Indian Statistical Institute, Bangalore

B. Math. First Year, First Semester

Probability Theory: Final Examination

Date : 03-12-2012 Time: 3 hours

Maximum score: 100

Weightage: 60

1. Consider a box with 10 tokens numbered from 1 to 10. You select three tokens at random one after another without replacement. What is the probability that the numbers you select are in increasing order? Given that they are in increasing order what is the conditional probability that the last number picked is 5? [10]

2. Fix $0 < p < 1$. Suppose X, Y are two independent identically distributed geometric random variables having distribution:

$$P(X = n) = P(Y = n) = q^{n-1}p \quad \text{for} \quad n \geq 1.$$

Take $Z = X - Y$. Find the distribution, expectation and variance of Z. [15]

3. A fair coin is tossed 5 times. Let F be the number of 'Heads' in first 3 tosses and let G be the total number of 'Heads' in 5 tosses. Write down the joint distribution of (F, G). Compute the conditional distribution of F given that $G = 3$. Compute the conditional expectation of F given that $G = 3$. [20]

4. Suppose U is a random variable having uniform distribution on the interval $[-1, 3]$. Find the distribution function and density for random variables R, S, T where $R = 5 - 2U$, $S = U^2$ and $T = U^3$. [20]

5. Suppose L is a random variable having exponential density with parameter $\lambda > 0$. Find the distribution and densities of $M = \frac{1}{L}$ and $N = |1 - L|$. [20]

6. State and prove Chebyshev's inequality for discrete random variables with finite second moment (If you need Markov's inequality, you should prove it). Use Chebyshev's inequality to get an upper bound for $P(G \geq 4)$, where G has normal distribution with mean 1 and variance 9. [20]

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Probability Theory: Backpaper Examination

Date : Time: 3 hours

Maximum score: 100

1. An urn contains 10 black balls, 6 red balls, and 4 white balls. Three balls are chosen at random with replacement from the urn. Let B be the number of black balls chosen and let R be the number of red balls chosen. Write down the joint distribution of (B, R). Compute the conditional distribution of B given $R = 1$. [20]

2. Let Y be a Poisson random variable with parameter $\lambda > 0$. Compute the moment generating function, expectation and variance of Y. [10]

3. Suppose class B-1 has 10 boys and 5 girls, class B-2 has 6 boys and 4 girls and class B-3 has 5 boys and 1 girl. One student is chosen at random. If it is a girl what is the probability that she is from B-1? [10]

4. Choose a point (X, Y) at random from the triangle in R^2 with vertices $(1, 1), (1, 4)$ and $(3, 0)$. Find distribution function, density and expectation of X. Find distribution function, density and expectation of $Z = X + Y$. [20]

5. Fix $0 < \sigma < \infty$. Suppose L is a random variable having normal density with mean 0 and variance σ^2. Find densities of $M = L^2$ and $N = |L|$. [20]

6. State and prove Weak Law of Large numbers for i.i.d. random variables with finite second moment. [20]